For each problem below, draw a diagram and write out all variables and equations before solving the problems. Do at least 3 the rest are bonus.

1. A marble with speed 1.5 m/s rolls off the edge of a table 4 m high. How long does it take the marble to hit the floor, and how far from the edge of the table does the marble hit the floor?

As soon as the marble is over the edge of the table, it's in free-fall, meaning that it begins accelerating straight down, under the influence of the Earth's gravitational acceleration, 9.8 m/s². This means, that even though it has the horizontal speed of 1.5 m/s, the time it takes to hit the floor can be found by basically ignoring this speed for the moment.

As soon as it leaves the edge of the table, it's vertical speed, \( v_{y0} \), and it must fall a distance 4 m before it hits the floor. How long will this take?

Use Equation 1: 
\[
y = y_0 + v_{y0}t + \frac{1}{2}at^2
\]
where \( y_0 = 4 \) m, \( y = 0 \) meters (the floor), \( v_{y0} = 0 \), and \( a = -9.8 \) m/s². Plugging these numbers in we get Equation 2:
\[
0 = 4 - \frac{1}{2} \cdot 9.8 \cdot t^2
\]
Solve for \( t \) to get Equation 3:
\[
t = \sqrt{\frac{2(4 \text{ m})}{9.8 \text{ m/s}^2}} = 0.9 \text{ seconds.}
\]
The marble will hit the floor in 0.9 seconds.

How far will the marble travel? Well, we just figured out that after 0.9 seconds, the marble is on the floor and its "flight" is over. So, to find how far from the edge of the table it traveled, we can use this equation:

Equation 4:
\[
x = x_0 + v_{x0}t + \frac{1}{2}at^2
\]
We'll set \( x_0 = 0 \), right at the table's edge, so \( x \) will be our landing distance. \( a \) in the equation is zero, since there is no horizontal acceleration on the marble. There is a vertical one (gravity), but we're analyzing horizontal motion now, so \( a = 0 \). What about \( v_{x0} \)? That's the 1.5 m/s, the speed at which it left the table. \( t \) here will be the 0.9 seconds from above. This is the total time the marble has to fly. So plugging in the numbers, we'll get Equation 5:
\[
x = 1.5 \text{ m/s} \cdot 0.9 \text{ seconds}
\]
or, the marble will land 1.35 meters from the edge of the table.

2. A ball is shot upward from the level ground at an angle of 75 degrees with respect to the horizontal. It is given an initial speed of 100 m/s. How long will it take before the ball hits the ground?

The reason the ball hits the ground after being launched is because of the Earth's gravity, 9.8 m/s². This is a strictly vertical acceleration, which only affects vertical-directed velocities. So the focus in this problem is how gravity interacts some vertical velocity.

In this case the ball has a velocity which is pointed at 75 degrees from the ground. This gives it two components of velocity, one horizontal (parallel to the ground), and one vertical (perpendicular to the ground).

The horizontal component, \( v_{x0} \), is Equation 1:
\[
v_{x0} = 100 \text{ m/s} \cos 75
\]
and the vertical component, \( v_{y0} \), is Equation 2:
\[
v_{y0} = 100 \text{ m/s} \sin 75 \text{ or } v_{y0} = 96.59 \text{ m/s}
\]
As stated, we only need to be concerned with \( v_{y0} \) since its the only component of the velocity that will be affected by the Earth's gravity.

So what happens? Well, part of the ball's motion consists of it being launched straight upward with a velocity of 96.59 m/s. You can find this time using this equation:

Equation 3:
\[
y = v_{y0}t + \frac{1}{2}at^2; \quad y = 0
\]
so:
\[
-v_{y0} = \frac{1}{2}at
\]
\( a = -g \) so:
\[
-v_{y0} = \frac{1}{2} \cdot -9.8
\]
Equation 4:
\[
t = \frac{2v_{y0}}{g}
\]
so after being launched, the ball will hit the ground again in
3 A cannon ball is shot from a cannon with a speed of 150 m/s. The cannon is pointing above
the ground at an angle of 30 degrees. The ground is level everywhere around the cannon.
How far from the cannon will the ball land?

Since the ground is level, we can use the range equation to solve this problem. The equation is
Equation 1: \[ R = \frac{v_0^2 \sin(2\theta)}{g} \]
Here, \( \theta = 30 \) degrees and \( v_0 = 150 \text{ m/s} \). \( g = 9.8 \text{ m/s}^2 \). Plugging in the numbers, we get
Equation 2: \[ R = \frac{(150 \text{ m/s})^2 \sin(2 \times 30 \text{ degrees})}{9.8 \text{ m/s}^2} \]
The cannon ball will land 1988.32 meters from the cannon.
Note that using the range equation (Equation 1) only works if the launch position is at the same level as the
landing position. It is absolutely wrong to use it in any

4 A soccer goalie punts a soccer ball. The ball leaves the goalie's foot 1.2 meter above the
ground, at an angle of 50 degrees, with a speed of 35 m/s. The angle is measured with
respect to the flat, horizontal ground. How far does the ball travel before hitting the
ground?

You cannot use this formula
Equation 1: \[ R = \frac{v_0^2 \sin(2\theta)}{g} \]
in this problem since the ball was launched 1.2 meter above the ground.
To start with, let's find the x and y components of the launch velocity, \( v_x = 35 \text{ m/s} \)
like this (we'll need them):
Equation 1 and 2: \[ v_x = 35 \text{ m/s} \cos(50) \text{ and } v_y = 35 \text{ m/s} \sin(50) \]
or \[ v_x = 22.49 \text{ m/s} \] \[ v_y = 26.81 \text{ m/s} \]
Ok, to start solving this problem, let's first see how long the ball is in the air.
This time governs how long the ball has to fly and reach the distance at which
it will eventually land.
The flight time is governed only by \( v_y \) (the vertical component of velocity),
and \( g = 9.8 \text{ m/s}^2 \), the Earth's acceleration due to gravity.
We can use this equation
Equation 3: \[ y = v_y t + \frac{1}{2} a t^2 \]
Here, we'll set \( y = 0 \) (the ground), \( v_y = 35 \text{ m/s} \), \( v_y = 26.81 \text{ m/s} \), and \( a = g = -9.8 \text{ m/s}^2 \). Plugging in the numbers, we get
\[ 0 = 26.81 \text{ m/s} t + \frac{1}{2} (-9.8 \text{ m/s}^2) t^2 \]
This is a quadratic equation, so we'll use the quadratic formula:
Equation 4: \[ t = \frac{-26.81 \pm \sqrt{(26.81)^2 - 2(-9.8 \text{ m/s}^2)(1.2 \text{ meter})}}{-9.8 \text{ m/s}^2} \]
This means, all told, the ball will be in the air for 5.5 seconds.
Finally, we can figure out how far from the kick the ball lands using this
equation
Equation 5: \[ x = v_x t - 0.5 a t^2 \]
where \( t = 5.5 \text{ seconds} \), and \( v_x = 22.49 \text{ m/s} \).
Calculating this out, you'll get that ball will land 123.69 meters from the spot.
5 An enemy ship is anchored in the sea 975 m from a island defending itself. The maximum velocity a defense cannon fire a cannon ball is 180 m/s. At what angle, with respect to sea level, should the cannon be elevated to hit the enemy ship? And, how far away from the island should the ship move to be out of range of the island’s defense cannon?

To answer these two questions, we’ll use this formula

Equation 1: \[ R = \frac{v_0^2 \sin(2\theta)}{g} \]

which says that a projectile (like the cannon ball), will land a distance \( R \) from where it was shot, given that its launch speed was \( v_0 \) and it was launched at an angle with respect to the level ground. \( g = 9.8 \text{ m/s}^2 \) for a projectile on the Earth.

To answer the first question, we’ll solve Equation 1 for \( \theta \) to get

Equation 2: \[ 2\theta = \sin^{-1} \left( \frac{gR}{v_0^2} \right) \quad \text{or} \quad \theta = \frac{1}{2} \sin^{-1} \left( \frac{gR}{v_0^2} \right) \]

In order to hit the ship, the cannon ball must have a range of \( R = 975 \text{ m} \). If the cannon ball is launched at maximum velocity, \( v_0 = 180 \text{ m/s} \). \( g = 9.8 \text{ m/s}^2 \). Plugging in these numbers, we get

Equation 4: \[ \theta = \frac{1}{2} \sin^{-1} \left( \frac{9.8 \text{ m/s}^2 \times 975 \text{ m}}{180 \text{ m/s}^2} \right) \]

or \( \theta = 8.57 \text{ degrees} \). The cannon ball would have to be launched at 8.57 degrees in order to hit the ship.

For the next question, we have to use the fact that all things being equal, a projectile will travel the largest distance when aimed at a 45 degree angle with respect to the level ground.

So, let’s plug \( \theta = 45 \text{ degrees} \) into Equation 1, along with \( v_0 = 180 \text{ m/s} \) and \( g = 9.8 \text{ m/s}^2 \).

Equation 5: \[ R = \frac{(180 \text{ m/s})^2 \times 2 \sin(2 \times 45)}{g} \]

or \( R = 3306.12 \text{ meters} \). The ship would have to move at least 3306.12 meters away from the island to avoid being hit by a cannon ball.

6 A person is trapped in the snow, and a rescue plane wants to drop them some emergency supplies. The plane is flying level at an altitude of 5000 meters at a speed of 500 m/s. How far in front of the person should the pilot drop the supplies, so that they land right on top of the trapped person?

The instant the rescue supplies are dropped from the airplane, they go into “free-fall,” which means they are accelerated toward the Earth due to the Earth’s gravity, 9.8 m/s².

When the supplies are released from the plane, they will have a horizontal component of velocity, which is the same as the plane, or 500 m/s . The vertical (up/down) component of the supply’s velocity, however is zero. Also, since the plane and supplies have the same horizontal velocity, the supplies will always stay directly below the plane as they fall (assuming, as always, there’s no air resistance).

Let’s calculate how long it will take the supplies to hit the ground. Use this equation

Equation 1: \[ y = y_0 + v_{0y} t + \frac{1}{2} a_y t^2 \]

Into this equation we’ll put if \( y_0 = 5000 \text{ meters} \), \( y = 0 \text{ meters} \), \( v_{0y} = 0 \text{ m/s} \) and \( a_y = -9.8 \text{ m/s}^2 \), then plugging into Equation 1, we get

Equation 2: \[ 0 \text{ meters} = (5000 \text{ meters}) + 0 \times t + \frac{1}{2} (-9.8) t^2 \]

\((y=0 \text{ means ground level})\).

Equation 3: Solve for \( t \) to get: \[ t = \sqrt{\frac{5000 \text{ meters}}{9.8 \text{ m/s}^2}} \] or \( t = 31.94 \text{ seconds} \). It will take the supplies 31.94 seconds to hit the ground.

Now, in 31.94 seconds , the plane travels

Equation 4: \[ x = v_{0x} t \quad [x = x_0 + v_{0x} t - \frac{1}{2} a_x t^2] \quad x_0 = 0; \quad a_x = 0 \]

or \( x = 15970 \text{ meters} \). Plugging in if \( v_{0x} = 500 \text{ m/s} \) and \( t = 31.94 \text{ seconds} \), then plugging into Equation 4, we get

Equation 5: \[ x = (500 \text{ m/s}) \times (31.94 \text{ seconds}) \]

or \( x = 15970 \text{ meters} \).

This means that from the time of release to the time the supplies hit the ground, the plane will travel 15970 meters.

So, the pilot should release the supplies at a distance of 15970 meters in
7 You are playing handball, and throw a ball with a speed of 20 m/s at an angle of 25 degrees above the horizontal directly at a wall. The wall is 100 m from the release point. a) How long does it take before the ball hits the wall? b) How far up the wall does the ball hit?

8 A ball is thrown upward at an angle of 60 degrees and lands on the top edge of a building that is 200 m away. The top edge of the building is 50 m above the throwing point. How fast